DESTROYING A GLASS-LIKE MATERIAL IN THE HYPERSONIC FLOW OF A GAS WITH CONSIDERATION OF RADIANT HEAT TRANSFER WITHIN THE MATERIAL

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UDC 536.422.1

We examine the destruction of a material with the formation of a liquid film. Radiation within the material is taken in approximation of radiant heat transfer. We have derived a final formula which relates the entrainment in the gaseous phase with the decisive parameters of the problem.

The problem of destroying a glass-like material in the hypersonic flow of a gas reduces to the simultaneous solution of the equations for a multicomponent boundary layer and the equations for a viscous liquid glass film and for the joining of these solutions at the front of the destruction. Reference [1] gives these equations for the vicinity of the critical point and line. In the solution of the liquid-film equations it is usual to neglect the radiant transfer of heat within the coating. The radiation within the body is taken into consideration in [5]. However, here it is only the heat-conduction equation and the radiation heat-transfer equation that are solved numerically here, with no consideration given to the equation of motion.

If we consider the radiation within the material, the equations for the nonsteady boundary layer in the case of the melt film in the vicinity of the critical point and line will differ from the corresponding equations of the paper by Tirskii only in connection with the term $-\partial H_y/\partial y$ in the energy equation. Here H_y denotes the radiant energy flux along the y-axis, directed along the normal to the surface.

In the approximation of radiant heat transfer [2] we have

$$H_y = -\frac{16}{3}\frac{\sigma_r T_1^3}{\rho_1 \alpha_r}\frac{dT_1}{dy} = -\frac{16}{3} l_r \sigma_r T_1^3 \frac{dT_1}{dy}. \tag{1}$$

We will assume that the viscosity of the glass-like material is a function of temperature in the following manner:

$$\begin{aligned} &\mu_1 = \mu^* \exp(T^*/T_1) \text{ when } T_1 > T^{**}, \\ &\mu_1 = \infty \qquad\qquad \text{when } T_1 \leqslant T^{**}, \end{aligned} \tag{2}$$

i.e., below a temperature T** the glass is a solid, and μ^*, T*, and T** are constants.

Repeating the transformations and evaluations of [1], we find the dimensionless equations of motion for the liquid film, in conjunction with the radiation:

$$\frac{d^2\varphi_1}{d\eta_1^2} = -(\tau_0 + \eta_1)\exp\left(-\frac{1}{\theta_1}\right), \tag{3}$$

$$\frac{d}{d\eta_1}\left[\left(1 + \lambda_R \theta_1^3\right)\frac{d\theta_1}{d\eta_1}\right] + \varphi_1 \frac{d\theta_1}{d\eta_1} = 0 \tag{4}$$

with the boundary conditions

$$\begin{aligned} &\varphi_1(0) = \alpha_1, \quad \theta_1(0) = \theta_0 = T_w/T^*, \\ &\frac{d\varphi_1(\infty)}{d\eta_1} = 0, \quad \theta_1(\infty) = \theta_m = T_m/T^*. \end{aligned} \tag{5}$$

Lomonosov Mechanics Institute, Moscow State University, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 17, No. 1, pp. 150-154, July, 1969. Original article submitted October 1, 1968.

Here $\lambda_R = 16/3\sigma_r T^{*3}/\lambda_1\rho_1\alpha_r$ is a new dimensionless parameter that is a function of the absorbtion factor. The coefficient α_r, generally speaking, is a function of the temperature and of the properties of the material. But since we have very little information about this quantity, we will assume in the following that α_r = const.

Initially we will solve (3) and (4) in approximate terms. We note that when η_1 varies from 0 to ∞, there is a reduction in the value of θ_1 from θ_0 to θ_m, while the function φ_1 increases from α_1 to some constant which we denote d (it represents the dimensionless total entrainment). The change in the function φ_1 takes place in a very narrow layer, and we will therefore comit no great error by replacing φ_1 in (4) by d = const. Equation (4) can then be solved separately. Integrating that equation twice, in conjunction with (5) we will derive the formula which relates η_1 with θ_1:

$$-\eta_1 = \frac{1+\lambda_R\theta_m^3}{d}\ln\frac{\theta_1-\theta_m}{\theta_0-\theta_m} + \frac{\lambda_R}{d}\left\{\frac{1}{3}\left[(\theta_1-\theta_m)^3-(\theta_0-\theta_m)^3\right] + \frac{3}{2}\theta_m\left[(\theta_1-\theta_m)^2-(\theta_0-\theta_m)^2\right] + 3(\theta_1-\theta_0)\,\theta_m^2\right\}. \quad (6)$$

From (6) we can find the approximate relationship between θ_1 and η_1 which is in good agreement with (6) for small η_1. We will seek this relationship in this form

$$\theta_1-\theta_m = (\theta_0-\theta_m)\exp(-p_1\eta_1-p_2\eta_1^2). \quad (7)$$

In this case

$$p_1 = d/(1+\lambda_R\theta_0^3),\quad p_2 = 1.5\,d^2\lambda_R\theta_0^2(\theta_0-\theta_m)/(1+\lambda_R\theta_0^3)^3.$$

We will approximate the viscosity formula (2) in accordance with Beta and Adams [3]

$$\exp\left(-\frac{1}{\theta_1}\right)\approx\exp\left(-\frac{1}{\theta_0}\right)\left(\frac{\theta_1-\theta_m}{\theta_0-\theta_m}\right)^{(\theta_0-\theta_m)/\theta_0^2}. \quad (8)$$

Using (7) and (8) we will transform (3) to

$$\frac{d^2\varphi_1}{d\eta_1^2} = -(\tau_0+\eta_1)\exp\left(-\frac{1}{\theta_0}\right)\exp\left(-p_1\frac{\theta_0-\theta_m}{\theta_0^2}\eta_1 - p_2\frac{\theta_0-\theta_m}{\theta_0^2}\eta_1^2\right).$$

We will integrate the last equation twice, replacing the probability integral $\Phi(z) = 2/\sqrt{\pi}\int_0^z\exp(-t^2)dt$ by the following asymptotic expansion [4]:

$$\frac{\sqrt{\pi}}{2}\left[1-\Phi(z)\right] = \frac{\exp(-z^2)}{2z}\left\{1-\frac{1}{2z^2}+\frac{1\cdot3}{(2z^2)^2}-\frac{1\cdot3\cdot5}{(2z^2)^3}+\ldots\right\}.$$

If we bear in mind that $\varphi_1(0) = \alpha_1$, $\varphi_1(\infty)$ = d, we derive the final formula

$$\alpha_1 = d - \left[\tau_0\left(1-9\theta_0\frac{\lambda_R\theta_0^3}{1+\lambda_R\theta_0^3}\right) + \frac{2\theta_0^2(1+\lambda_R\theta_0^3)}{(\theta_0-\theta_m)d}\left(1-18\theta_0\frac{\lambda_R\theta_0^3}{1+\lambda_R\theta_0^3}\right)\right]\exp\left(-\frac{1}{\theta_0}\right)\frac{\theta_0^4(1+\lambda_R\theta_0^3)^2}{(\theta_0-\theta_m)^2d^2}. \quad (9)$$

To evaluate the accuracy we will compare several calculations accomplished with this formula with the numerical solutions from (3) and (4). To avoid solving the boundary-value problem with the boundary conditions (5) we will proceed from the asymptotic curve, bearing in mind that for large θ_1 we have $\eta_1\varphi_1$ = d, while θ_1 and $d\theta_1/d\eta_1$ are associated with η_1 by relationship (6) in which θ_0 must be replaced by some constants. Then, having specified the value for $\eta_1 = \eta_1^*$, we replace conditions (5) by the following:

$$\varphi_1(\eta_1^*) = d,\quad \theta_1(\eta_1^*) = \theta_1^* > \theta_m,$$
$$\frac{d\varphi_1(\eta_1^*)}{d\eta_1} = 0,\quad \frac{d\theta_1(\eta_1^*)}{d\eta_1} = -\frac{\theta_1^*-\theta_m}{1+\lambda_R\theta_1^*}d. \quad (10)$$

The value of θ_1^* has been chosen so that for η_1^* the right-hand member of (3) is equal to zero. This eliminates the need of assigning a specific value to T** (see (2)). Solving the problem with conditions (10) at the right-hand end, at the left-hand end we derive certain values of $\varphi_1(0) = \alpha_1$ and $\theta_1(0) = \theta_0$. With a change in η_1^* these values will change. It is thus possible to construct curves showing α_1 as a function of θ_0 with the values of the remaining parameters fixed. For θ_m = 0.008, d = 10^{-3}, τ_0 = 1000 these curves are shown in Fig. 1. For purposes of comparison, here we also find the curves corresponding to formula (9). As we can see, the deviations from the exact value are insignificant. Having the solution for the liquid film, we can combine it with the solution for the gas phase.

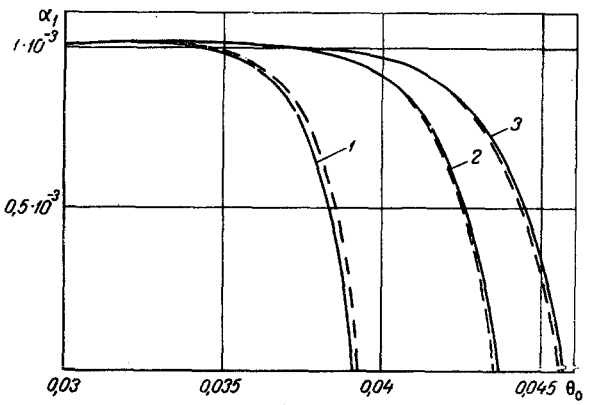

Fig. 1. The dimensionless entrainment in the gas phase as a function of the dimensionless temperature. The solid lines denote computer calculation while the dashed lines denote calculation in accordance with formula (9): 1) $\lambda = 10^5$; 2) 10^4; 3) 0.

Let us consider the problem of the suitability of the approximation of radiant heat transfer to the problem of destroying a glass-like material with formation of a liquid film. Expression (1) for the flux of radiant energy is valid only in the case of weak radiation anisotropy [2]. If we are calculating for a point deep within the material, at a distance greater than l_r, and if l_r is smaller than the distance at which the temperature difference is substantial, the anisotropy is small. At the surface the anisotropy may not be small and the actual temperature distributions within a layer of order l_r may differ somewhat from that calculated. However, since the viscosity of the glass depends significantly on the temperature, even a slight difference in temperature may yield a significant error in the determination of the entrainment in the liquid phase. Our approximation is therefore doubtlessly applicable only to the case in which l_r is smaller by an order of magnitude than the thickness of the liquid film. However, even for such small l_r failure to account for the radiation may lead to significant error. For example, if the film thickness is 1mm, we have $l_r = 0.1$ mm, and the thermophysical parameters are the same as in the case of quartz glass [1], so that $\lambda_R = 10^4$, and unlike the case without radiation ($\lambda_R = 0$) it is rather large, as we can see from Fig. 1.

Calculation of the rate of entrainment for the quartz glass for the most intensively heated point of a typical reentry trajectory at escape velocity yields the following results. Consideration of the radiation when $l_r = 0.1$ mm ($\lambda_R = 10^4$) increases the rate of entrainment by 22%, while for $l_r = 1$ mm ($\lambda_R = 10^5$) the corresponding figure is 2.84. In this case the entrainment as a consequence of varporization virtually does not change, but there is a very substantial increase in the entrainment in the liquid phase.

The literature contains virtually no information on the absorbtion factor in glasses at high temperatures. Here we require basic experimentation to fill this gap. However, we will cite the data from [6]. For glasses with varying iron contents we have $l_r = 0.05$–10 cm.

It is obvious that for coatings of pure quartz glass the approximation of radiant heat transfer is useless. However, coatings which contain resins (glass, Textolite) in addition to the quartz may exhibit such small l_r that the use of this approximation is justified.

Moreover, numerical calculations of the equations of radiation and heat transfer from [5] – with consideration, however, of the radiation from the impact layer – show that for certain values of the parameters the path of the radiation l_r is no smaller than the thickness of the liquid film, while the layer with noticeable anisotropy of radiation is substantially smaller than this thickness, since the anisotropy diminishes as a consequence of the fact that the rays are reflected from the liquid–gas surface, as well as in the presence of radiation from the impact layer. In these cases our approximation is also valid.

NOTATION

y	is the coordinate;
H_y	is the radiant energy flux;
T_1	is the temperature in the film;
ρ_1	is the film density;
α_r	is the frequency-averaged absorbtion factor;
σ_r	is the Stefan–Boltzmann constant;
l_r	is the mean free path of the radiation;
μ_1	is the viscosity of the film;
μ^*, T^*, and T^{**}	are constants in formula (2);
φ_1	is a dimensionless stream function;
$\theta_1 = T_1/T^*$	is the dimensionless temperature;

η_1 is a dimensionless coordinate directed perpendicular to the surface of the material;
τ_0 is the dimensionless friction at the surface;
λ_R is a dimensionless parameter which is a function of the absorbtion factor;
$\theta_0 = T_W/T^*$;
$\theta_m = T_m/T^*$;
T_W is the temperature at the film surface;
T_m is the temperature within the depth of the material;
d is the total dimensionless entrainment;
α_1 is the dimensionless entrainment in the gas phase;
p_1 and p_2 are parameters in formula (7);
z is a variable in the probability integral;
$\Phi(z)$ is the probability integral.

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